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Rigorous solution of unsteady forced convection heat transfer

HSIAO-TSUNG LIN and WEN-TONG CHENG

Department of Chemical Engineering, National Central University, Chungli, Taiwan 320, Republic of China

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1. INTRODUCTION

The fundamental problem of unsteady-state heat transfer between solid surfaces and steady forced flows has long been the subject of many investigations [1–6]. However, all the previous series expansion solutions (refs. [1–3] and the literature cited therein), local similarity and nonsimilarity solutions [4, 5], and even the finite-difference solution [6] are approximate ones. Only the solutions in the initial and final stages of transient have been verified by comparison with the unsteady conduction and steady convection solutions. The accuracy of these approximate solutions in the transition stage remains uncertain. There is still a need for a simple and very effective solution method that will give precise solutions over the entire transient history of unsteady convection.

In the present study, we introduce a new method for analyzing unsteady forced convection heat transfer. The method is based on the concept that the whole transient history consists of the initial stage of unsteady conduction, the final stage of steady convection, and the transition stage between these two limiting cases. Our approach is to model the thermal boundary-layer thickness of the unsteady convection as an appropriate combination of those of unsteady conduction and steady convection. In addition, a proper dimensionless time is proposed as the ratio of the thermal boundary-layer thickness of unsteady convection to that of steady convection. As a result, the transformed energy equation describes accurately the entire transient history and can be reduced readily to the conventional similarity equations of unsteady conduction and steady forced convection. Therefore, very precise finite-difference solutions and a simple correlation equation can be obtained for $0.001 \leqslant Pr \leqslant \infty$.

We demonstrate the proposed solution method for unsteady forced convection heat transfer with the case of a rotating disk.

2. ANALYSIS

The fluid of the steady laminar flow induced by a rotating disk is assumed to be incompressible and with constant properties. Initially the fluid and the solid surface are at the same temperature T_∞. At a certain instant the surface temperature is changed from T_∞ to T_0 and maintained thereafter. This situation is the case of a step change in surface temperature. Another case considered is a step change in heat flux from 0 to q_0.

The energy equation of unsteady heat transfer from the suddenly heated surface to the steady laminar flow can be written as

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}=\alpha\frac{\partial^2 T}{\partial z^2} \tag{1}$$

where u and w are the velocity components in the radial and axial coordinates, respectively. This equation is subject to the initial and boundary conditions

$$T(r,z,0)=T_\infty \tag{2}$$

$$T(r,\infty,t)=T_\infty. \tag{3}$$

NOMENCLATURE

F	dimensionless radial velocity, $u/\omega r$
G	dimensionless tangential velocity, $v/\omega r$
h	local heat transfer coefficient [J s^{-1} m^{-2} K^{-1}]
H	dimensionless axial velocity, $w\delta/\alpha$
k	thermal conductivity of fluid [J s^{-1} m^{-1} K^{-1}]
Nu	Nusselt number, hr/k
Pr	Prandtl number, ν/α
q_0	heat flux at the disk surface [J s^{-1} m^{-2}]
r	radial coordinate [m]
Re	Reynolds number, $\omega r^2/\nu$
t	time [s]
T	temperature [K]
u, v, w	velocity components in radial, azimuthal, and axial directions [m s^{-1}]
z	axial coordinate [m].

Greek symbols

α	thermal diffusivity of fluid [m^2 s^{-1}]
δ	thermal boundary-layer thickness of unsteady convection, $(\delta_t^{-1}+\delta_s^{-1})^{-1}$ [m]
δ_s	thermal boundary-layer thickness of steady forced convection, $\sim r/(\sigma Re^{1/2})$ [m]
δ_t	penetration thickness of transient conduction, $(4\alpha t)^{1/2}$ [m]
ζ	dimensionless axial coordinate, z/δ
η	conventional dimensionless axial coordinate, $(z/r)Re^{1/2}$
θ	dimensionless temperature, $(T-T_\infty)/(T_0-T_\infty)$
ν	kinematic viscosity of fluid [m^2 s^{-1}]
ξ	dimensionless time, δ/δ_s
σ	$Pr/(1+Pr)^{2/3}$
ϕ	dimensionless temperature, $(T-T_\infty)/(q_0\delta/k)$
ω	angular velocity of rotating disk [rad s^{-1}]
$\mathbf{1}(t)$	Heaviside unit step function, $=1$ for $t>0$; $=0$ for $t \leqslant 0$.

Subscripts

s	steady forced convection
t	transient conduction
0	at the disk surface
∞	far from the disk surface.

The boundary conditions at the disk surface is

$$T(r,0,t) = T_\infty + (T_0 - T_\infty)\mathbf{1}(t) \tag{4}$$

for the case of a step change in temperature; or

$$-k\left(\frac{\partial T}{\partial z}\right)_{z=0} = q_0\,\mathbf{1}(t) \tag{5}$$

for a step change in heat flux. The function $\mathbf{1}(t)$ in equations (4) and (5) is the Heaviside unit step function: $\mathbf{1}(t)$ is equal to 1 for $t>0$; and 0 for $t \leqslant 0$.

In the initial stage of the whole transient history, the heat transfer is basically a heat diffusion process with a penetration thickness $\delta_t \sim (4\alpha t)^{1/2}$, while, in the final stage, it is essentially a steady forced convection, the scale of thermal boundary-layer thickness of steady convection is

$$\delta_s \sim \frac{r}{\sigma Re^{1/2}} \tag{6}$$

where $\sigma = Pr/(1+Pr)^{2/3}$ [7] and the Reynolds number $Re = \omega r^2/\nu$. The addition of $\sigma(Pr)$ properly scales the thermal boundary-layer thickness for large and small Prandtl numbers from 0.001 to infinity.

It is obvious that the heat transfer regime δ grows as δ_t at the initial stage, and finally develops to δ_s at the final stage of steady forced convection. We therefore assume that the heat transfer regime δ would be an appropriate association of δ_t and δ_s. We propose that

$$\frac{1}{\delta} = \frac{1}{\delta_t} + \frac{1}{\delta_s}. \tag{7}$$

At the initial stage, $\delta_t \ll \delta_s$ and equation (7) reduced to $\delta = \delta_t$, while, at the final stage, $(4\alpha t)^{1/2} > \delta_s$ and $\delta \to \delta_s$.

For the pseudo-similarity transformation of the energy equation, we define a dimensionless axial coordinate in terms of the heat transfer regime δ:

$$\zeta = \frac{z}{\delta} = \frac{z}{r}\left(\frac{r}{\sqrt{4\alpha t}} + \sigma Re^{1/2}\right). \tag{8}$$

Moreover, a dimensionless time is proposed as the ratio of the thermal boundary-layer thickness of unsteady convection to that of steady convection:

$$\xi = \frac{\delta}{\delta_s} = \frac{\delta_t}{\delta_t+\delta_s} = \left(1+\frac{\delta_s}{\delta_t}\right)^{-1} = \left(1+\frac{r/(\sqrt{4\alpha t})}{\sigma Re^{1/2}}\right)^{-1}. \tag{9}$$

Note that $\xi \to 0$ for the case of unsteady conduction at very small times, whereas $\xi = 1$ for the case of steady convection in which $\delta = \delta_s$.

For the case of a step change in surface temperature, the dimensionless temperature is usually defined as $\theta = (T-T_\infty)/(T_0-T_\infty)$, while, for a step change in surface flux, a special nondimensional form of temperature is introduced:

$$\phi = \frac{T-T_\infty}{q_0\delta/k} = \frac{T-T_\infty}{q_0 r/k}\left(\frac{r}{\sqrt{4\alpha t}} + \sigma Re^{1/2}\right). \tag{10}$$

In addition, a nondimensional form of the axial velocity is defined as

$$H = \frac{w\delta}{\alpha} = \frac{wr}{\alpha}\left(\frac{r}{\sqrt{4\alpha t}} + \sigma Re^{1/2}\right)^{-1} = \frac{wr/\alpha}{\sigma Re^{1/2}}\xi. \tag{11}$$

Using the dimensionless variables defined above, the energy equation (1) can be transformed to yield

$$\theta'' - H\theta' + 2(1-\xi)^3\zeta\theta' = 2\xi(1-\xi)^3\frac{\partial\theta}{\partial\xi} \tag{12}$$

for the case of a step change in surface temperature. The primes in this equation designate partial derivatives with respect to ζ. The initial condition and the boundary condition at $z \to \infty$ (i.e. $z > \delta$) are combined into $\theta(0,\infty) = 0$, while the boundary condition at $z = 0$ becomes $\theta(\xi, 0) = 1$.

Equation (12) can be readily reduced, by setting $\xi = 0$, to the similarity equation of unsteady conduction:

$$\theta'' + 2\zeta\theta' = 0. \tag{13}$$

Note that the term $H\theta'$ has been eliminated from equation (12), since H is proportional to ξ. The energy equation (12) is also reducible to the following similarity equation of steady forced convection by setting $\xi = 1$:

$$\theta'' - H\theta' = 0. \tag{14}$$

For the case of a step change in surface heat flux, the transformed energy equation and initial and boundary conditions are

$$\phi''-H\phi'+2(1-\xi)^3(\zeta\phi'-\phi) = 2\xi(1-\xi)^3\frac{\partial\phi}{\partial\xi} \quad (15)$$

$$\phi(0,\infty) = 0 \quad \phi'(\xi,0) = -1. \quad (16)$$

The similarity energy equation reduced from equation (15) for the limiting case of unsteady conduction ($\xi = 0$) is

$$\phi''+2\zeta\phi'-2\phi = 0, \quad (17)$$

while the similarity energy equation of steady convection ($\xi = 1$) is

$$\phi''-H\phi' = 0. \quad (18)$$

In the numerical solution of the transformed energy equations (12) and (15), the dimensionless transverse velocity $H(\zeta)$ had been obtained from the following similarity forms of the continuity and momentum equations

$$PrH'+2(1+Pr)^{4/3}\xi^2F = 0 \quad (19)$$

$$Pr^2F'''-PrHF'-(1+Pr)^{4/3}\xi^2(F^2-G^2) = 0 \quad (20)$$

$$Pr^2G''-PrHG'-2(1+Pr)^{4/3}\xi^2FG = 0 \quad (21)$$

subject to the following boundary conditions:

$$F(0) = 0, \quad G(0) = 1, \quad H(0) = 0$$

$$F(\infty) = 0, \quad G(\infty) = 0. \quad (22)$$

where $F = u/r\omega$ and $G = v/r\omega$ are the dimensionless radial and axial velocities.

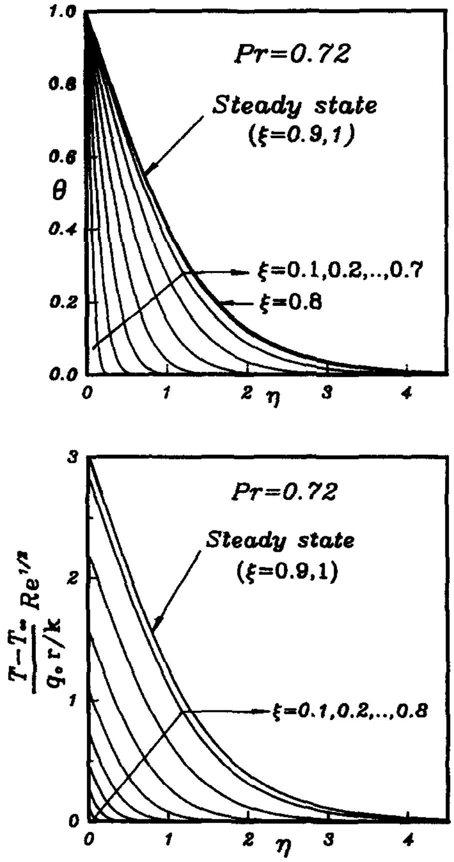

Fig. 1. The development of the dimensionless temperature profile following a step change in (a) surface temperature; (b) surface heat flux.

3. NUMERICAL METHOD

The nonsimilarity energy equations (12) and (15) were solved numerically by using a very effective finite-difference scheme, well known as the Keller's Box method [8].

The numerical integration started at $\xi = 0$ and marched step-by-step with $\Delta\xi = 0.01$ to $\xi = 1$. The step size of the ζ-coordinate, $\Delta\zeta$, and the edge of the boundary-layer, ζ_∞, are adjusted for different ranges of Pr. The uniform grid $\Delta\zeta$ has been chosen as 0.0002 for $Pr = 0.001$; 0.001 for $0.01 \leqslant Pr \leqslant 0.1$; and 0.01 for $Pr \geqslant 0.7$. The edge of the boundary layer was varied from $\zeta_\infty = 7$ for $Pr = 0.001$ to $\zeta_\infty = 32$ for $Pr \geqslant 10\,000$.

4. RESULTS AND DISCUSSION

4.1. *Temperature profiles*

The development of the dimensionless temperature profiles following a step change in surface temperature and heat flux is shown in Fig. 1. In this figure, we use the conventional dimensionless axial coordinate $\eta = (z/r)Re^{1/2} = (\xi/\sigma)\zeta$ instead of ζ to eliminate the time variable. Thus, the development of temperature profiles with time can be shown explicitly. For the same reason, the dimensionless temperature $(T-T_\infty)Re^{1/2}/(q_0r/k) = (\xi/\sigma)\phi$ is used in plotting Fig. 1(b) for the case of a step change in heat flux.

Figure 1 shows the step-by-step variation of thermal boundary layer when the dimensionless time variable ξ increases. For $\xi > 0.9$, the temperature profiles nearly coincide with that of the steady convection.

4.2. *Nusselt numbers*

For the case of a step change in surface temperature, the Nusselt number, $Nu = hr/k$, represents the heat transfer rate between the solid surface and the ambient fluid. Nu is related to the numerical results of $\theta'(\xi,0)$ by

$$\frac{Nu}{Re^{1/2}} = -\frac{\sigma}{\xi}\theta'(\xi,0) \quad (23)$$

whereas, for the case of a step change in heat flux, the Nusselt number represents the surface temperature and can be calculated from

$$\frac{Nu}{Re^{1/2}} = \frac{\sigma}{\xi\phi(\xi,0)}. \quad (24)$$

In Fig. 2, the present numerical solutions of $Nu/Re^{1/2}$ for the case of a step change in surface temperature are compared with the series expansion solutions [1] and the instant nonsimilarity solutions [5] for $Pr = 1$, 10 and 100. In this figure, we use the traditional time scale ωt which is related to the present dimensionless time by

$$\omega t = \frac{(1+Pr)^{4/3}}{4Pr}\left(\frac{\xi}{1-\xi}\right)^2. \quad (25)$$

This figure reveals that, at the initial and final stages, the previous approximate solutions are in excellent agreement with the finite-difference solution. However, at the transition stage the approximate solutions are somewhat overestimated. The maximum deviations of the approximate solutions from the finite-difference solutions over the whole time domain are also presented in Table 1.

Figure 3 presents the finite-difference solutions of $Nu/Re^{1/2}$ for both the cases of a step change in surface temperature and in heat flux. This figure shows that $Nu/Re^{1/2}$ decreases

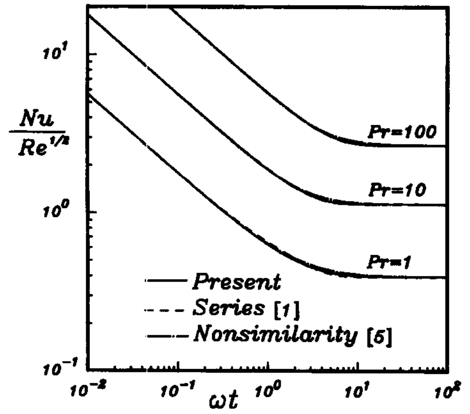

Fig. 2. Comparisons of the approximate solutions and the present finite-difference solutions of $Nu/Re^{1/2}$.

Table 1. Maximum deviations (percent) of the approximate solutions from the numerical solutions for a step change in surface temperature

Pr	1	10	100
Series solution	3.7	0.7	−0.4
Nonsimilarity solution	3.7	3.6	3.5

linearly with ωt at small times. This regime is regarded as the initial stage. At this stage, the numerical results coincide completely with the following exact solutions of unsteady conductions:

$$\frac{Nu_t}{Re^{1/2}} = \left(\frac{Pr}{\pi \omega t}\right)^{1/2} \tag{26}$$

for the case of a step change in surface temperature; and

$$\frac{Nu_t}{Re^{1/2}} = \frac{\sqrt{\pi}}{2}\left(\frac{Pr}{\omega t}\right)^{1/2} \tag{27}$$

for a step change in surface heat flux. The former was solved from equation (13) with the boundary conditions $\theta(0) = 1$ and $\theta(\infty) = 0$, while the latter was obtained from equation (17), subject to $\phi'(0) = -1$ and $\phi(\infty) = 0$.

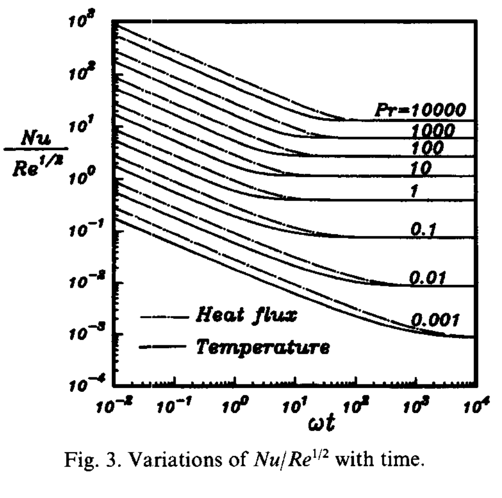

Fig. 3. Variations of $Nu/Re^{1/2}$ with time.

At the final stage, the numerical solutions of $Nu/Re^{1/2}$ for the cases of a step change in temperature and in heat flux are exactly the same and are in excellent agreement with the exact solution:

$$\frac{Nu_s}{Re^{1/2}} = -\sigma\theta'(1,0) = \frac{\sigma}{\phi(1,0)} = \frac{\sigma}{\int_0^\infty \exp\left[\int_0^\zeta H(\zeta)\,d\zeta\right] d\zeta}. \tag{28}$$

The exact solution for the case of uniform surface temperature was obtained from equation (14), associated with the boundary conditions $\theta(0) = 1$, $\theta(\infty) = 0$, while that for the case of uniform heat flux was solved from equation (18) with the boundary conditions $\phi'(0) = -1$, $\phi(\infty) = 0$.

4.3. *Correlation of Nusselt numbers*

A very comprehensive correlation equation for convenient estimation of the Nusselt number of unsteady convection is proposed as

$$\left[\frac{Nu}{\dfrac{r}{\sqrt{4\alpha t}} + \sigma Re^{1/2}}\right]^n = \left[(1-\xi)\frac{Nu_t}{r/\sqrt{4\alpha t}}\right]^n + \left[\xi \frac{Nu_s}{\sigma Re^{1/2}}\right]^n \tag{29}$$

which is based on the unsteady conduction and the steady convection solutions. The heat transfer group of transient conduction, $Nu_t/(r/\sqrt{4\alpha t})$, can be obtained by recasting the exact solutions of equations (26) and (27) as

$$\frac{Nu_t}{r/\sqrt{4\alpha t}} = \frac{2}{\sqrt{\pi}} \tag{30}$$

for the case of a step change in surface temperature; and

$$\frac{Nu_t}{r/\sqrt{4\alpha t}} = \sqrt{\pi} \tag{31}$$

for the case of a step change in heat flux. The numerical solutions of $Nu_t/(r/\sqrt{4\alpha t})$ at $\xi = 0$ are 1.12832 and 1.77245 for the cases of a step change in surface temperature and heat flux, respectively. These numerical results are in excellent agreement with the exact solutions.

A correlation equation of Nu_s for steady convection has been reported [8]:

$$\frac{Nu_s}{\sigma Re^{1/2}} = 0.6109\left(\frac{1+Pr}{0.5301 + 0.3996 Pr^{1/2} + Pr}\right)^{2/3}. \tag{32}$$

This correlation has a maximum error of 4% when compared with numerical data over the range of $0.001 \leqslant Pr \leqslant \infty$.

The exact solutions of $Nu_t/(r/\sqrt{4\alpha t})$ for unsteady conduction, equations (30) and (31), and the correlation equation (32) of $Nu_s/Re^{1/2}$ for steady forced convection was substituted into the correlation equation (29). The best fitting of the exponent n with the numerical data is listed in Table 2. The maximum error of this correlation over the entire time domain ($0 \leqslant \xi \leqslant 1$) is also presented in this table.

5. CONCLUSIONS

A very effective solution method has been developed for solving unsteady forced convection heat transfer between a steady laminar flow and solid surfaces, with a step change in temperature or heat flux. The method is based on the concept that the unsteady convection is a proper combination of the two limiting cases of unsteady conduction and steady convection processes. From this viewpoint, we have proposed an appropriate thermal boundary-layer thickness of unsteady convection in terms of the penetration depth of

Table 2. Values of n and the maximum error of the correlation equation (29) for the whole time domain

Range of Pr	n	Maximum error (%)
Step change in temperature:		
$0.01 \leqslant Pr \leqslant 0.6$	2.09	7.41
$0.7 \leqslant Pr \leqslant 10\,000$	3.68	6.52
Step change in heat flux:		
$0.01 \leqslant Pr \leqslant 0.6$	2.21	7.65
$0.7 \leqslant Pr \leqslant 10\,000$	3.85	5.92

transient conduction and the boundary-layer thickness of steady forced convection. With the growing boundary-layer thickness as a basis, we have introduced a nondimensional transverse coordinate and a dimensionless time, which are with proper scales for the initial, the transition, and the final stages of unsteady convection. Consequently, a comprehensive formulation and precise numerical solutions can be obtained over the entire transient history including unsteady conduction, true unsteady convection, and steady convection. Moreover, comprehensive and accurate correlation equations of Nusselt numbers have been developed, which are based on the solutions of unsteady conduction and steady convection.

The proposed method has been proved to be very effective and accurate via the demonstration of unsteady forced convection of a rotating disk. The present solution method has been applied successfully to many other unsteady convection heat transfer problems of various configurations and fluids.

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A unified correlation of laminar convective heat transfer from hot and cold circular cylinders in a uniform air flow

SHIN-HYOUNG KANG, KI-HYUCK HONG and SANGKEN KAUH
Department of Mechanical Engineering, Seoul National University, Seoul 151-742, Korea

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1. INTRODUCTION

The heat transfer on a circular cylinder embedded in a uniform cross flow is important not only due to its fundamental nature *per se* but also due to related engineering applications. Despite the simplicity of the relevant geometry, the flow over a cylinder frequently entails multi-faceted flow structures such as laminar boundary layer, transition, turbulent boundary layer, separation and wake formation. Fortunately, for the range of the Reynolds number pertinent to the hot-wire anemometry (commonly $Re \leqslant 40$), the flow is known to be steady and laminar. In this range of Re, the Nusselt number (dimensionless heat transfer coefficient) has been expressed as a canonical function of Pr and Re, with empirical correction factors accounting for the variation of fluid properties. Particularly for the case of air, the available heat transfer correlations are of the following form:

$$Nu = (A + B\,Re^n)(T_m/T_a)^p \tag{1}$$

due to a very weak dependency of Pr on temperature. However, a use of the mean temperature T_m in equation (1) is traditional rather than physically justifiable. A survey of the literature indicates that the existing correlations are of limited applicability to a narrow range of temperature ratios, typically for T_m/T_a smaller than 1.2. Furthermore, the available correlations are all biased, i.e. valid only for either hot ($T_w > T_a$) or cold ($T_w < T_a$) cylinders. Therefore, such hot